Visualizing the universe – part two

What a flood of e-mails I received after my previous lateral thoughts article "Visualizing the universe" – particularly from mathematicians! As you may recall, the article emphasized the scientific importance of being able to visualize in three dimensions (January 2004 p60). It posed four puzzles that I originally devised with Dame Kathleen Lonsdale for assessing her crystallography students; readers were invited to solve them mentally without using pencil or paper.

"Wearing a mathematical hat", says one reader Leon Firth, "you cannot prove a proposition to yourself or anyone else by experiment. You need to work in symbols or words to do that." Several others agree and pursue mathematical formulations for pages. Henry Ellington and others attempted to solve the puzzles by cutting, sticking and distorting various paper assemblies.

But many readers feel differently. "Words alone are the straitjacket of ideas," says James Prentice – as are numbers for many visual minds. An unexpected response came from the world of music via Philip Bishop – a player of "cerebrally demanding" jazz, who exercises and composes without an instrument, visualizing and manoeuvring the patterns of harmony, melody and rhythm. Another reader, Teedzai Nyamudeza in Zimbabwe, researches black holes in binary systems and eagerly asks if anyone can provide visual models for how they interact.

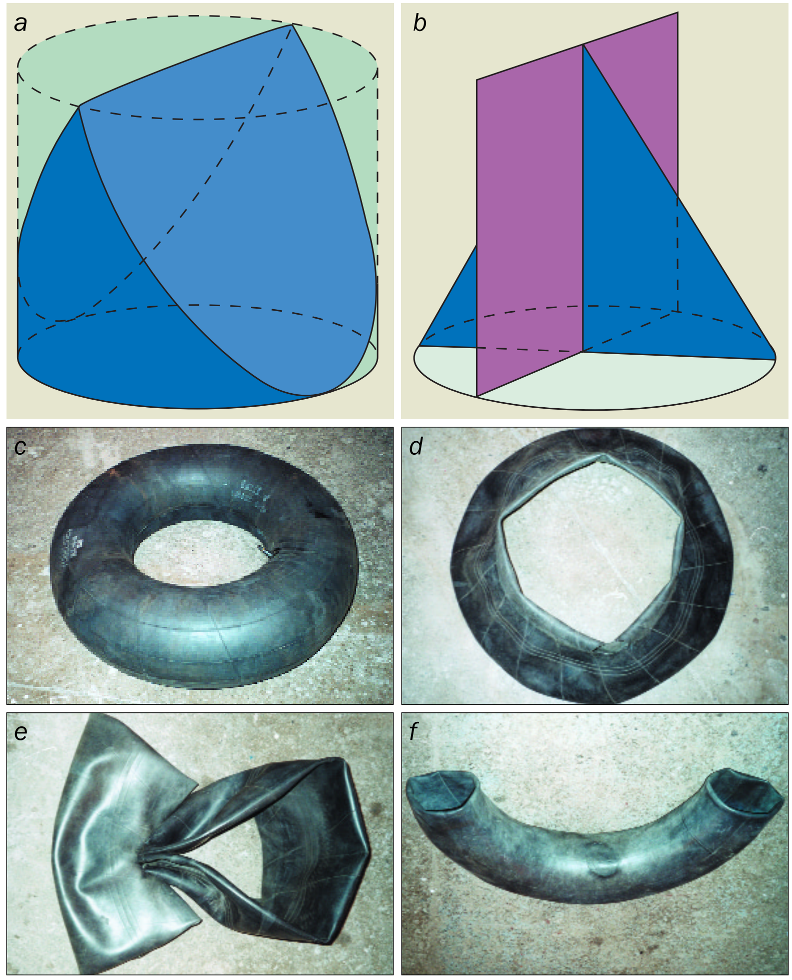
Many readers asked for pictorial explanations of the puzzles. The first envisaged a large, square car park with four cars, each starting simultaneously from each corner and chasing the car to its left. The answer to the question of where the cars will meet is simple – at the centre of the square. However, one mathematically inclined reader says that it depends on dimensions. Cars will meet because they are bulky but won't geometrical points go on spiralling into the centre forever?

The second question concerned a cube of cheese painted bright red and sliced through six times, twice in each dimension. The puzzle was dismissed as elementary by the few who mentioned it. One correspondent said that the cutting will create "27 cubes, of course", but failed to mention what colour their faces would be. Experience tells me that when asked, two-thirds of intelligent people dismiss the question, implying they cannot see it in their minds. But the visual mind has the answer in seconds.

Imagine the painted cheese cube cut into 27 minicubes, but still kept together. Each of the eight corner cubes has three red and three white faces. All 12 edges of the main cube have a central cube with two red and four white faces. Lastly, each of the faces of the main cube has a cube in the centre – so that's six cubes with one red face and five white. But that only accounts for 26 of the cubes. The 27th lies right in the centre and has six white faces.

Many readers were intrigued by the third problem, which concerned a flat steel plate with a circular hole of diameter *d*, a square hole with sides *d* and an isosceles triangle with base *d*. What single solid object will pass precisely through all three holes without a glimmer of light as it passes? For Sean Miller and other who are "stumped" the solution is shown in figure *a*.

Another reader, Tony Judd, imagines using a fretsaw to cut circular, square and triangular holes in the plate and then welding the offcuts together (figure *b*). This assembly will pass through all three holes although, as the thickness of the fretsaw blade has reduced the size a little, some light will be seen.

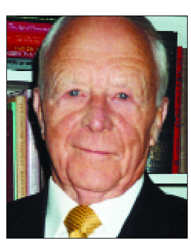


Words alone are the straitjacket of ideas

My final question prompted the biggest response. Yes, insist mathematicians, you can turn an inner tube inside out through a hole in the wall. Hans Gottlieb and Roger Phillips describe a hole first "expanded" round the circumference to leave two "handles" with a narrow bridge. They then fold back the handles and "shrink" the large opening back to a small hole. Well, that doesn't make sense in my practical world!

Fred Barclay – an experimentalist who sewed together the cuffs of his long-sleeved pullover to solve the puzzle – agrees with me. "I can't get it off inside out," he says. Kathleen and I also concluded that it is impossible to turn the tube inside out. What one obtains instead is a rather different yet symmetrical shape. Consider first the inflated inner tube that I begged from my local garage (figure *c*). Cut out the valve leaving a round hole. Start to pull through the inner wall (figure *d*). Things crinkle up but keep pulling (figure *e*). Now you can see where to pull in the right places and finally there is the double-walled semi-toroid (figure *f*).

To end, a reader quotes the mathematical historian Morris Kline, "The essence of any modern physical theory is a body of mathematical equations". Should I look forward to readers coming up with the full mathematical elucidation and proof of the inner-tube question?



Eric Voice is an independent consultant based in Thurso, Caithness, UK, e-mail ericvoice@tiscali.co.uk